IMC2018: Problems on Day 2

Problem 6. Let k be a positive integer. Find the smallest positive integer n for which there exist k nonzero vectors $v_1, \dots, v_k$ in $\mathbb{R}^n$ such that for every pair i, j of i with $|i - j| > 1$ the vectors v_i and v_j are orthogonal.

(Proposed by Alexey Balitskiy, Moscow Institute of Physics and Technology and I

Hint Sol

Problem 7. Let $(a_n)_{n=0}^{\infty}$ be a sequence of real numbers such that $a_0 = 0$ and

$$a_{n+1}^3 = a_n^2 - 8 \quad \text{for} \quad n = 0, 1, 2, \dots$$

Prove that the following series is convergent:

$$\sum_{n=0}^{\infty} |a_{n+1} - a_n|.$$

(Proposed by Orif Ibrogimov, National University of Uzbe

Hint Sol

Problem 8. Let $\Omega = (x, y, z) \in \mathbb{Z}^3 : y + 1 \geq x \geq y \geq z \geq 0$. A frog moves along the points of Ω by jumps of length 1. For every positive integer n, determi number of paths the frog can take to reach (n, n, n) starting from $(0, 0, 0)$ in exactly $3n$ jumps.

(Proposed by Fedor Petrov and Anatoly Vershik, St. Petersburg State Univ

Hint Sol

Problem 9. Determine all pairs $P(x), Q(x)$ of complex polynomials with leading coefficient 1 such that $P(x)$ divides $Q(x)^2 + 1$ and $Q(x)$ divides $P(x)^2 + 1$.

(Proposed by Rodrigo Angelo, Princeton University and Matheus Secco, PUC, Rio de Ja

Hint Sol

Problem 10. For $R > 1$ let $\square_R = (a, b) \in \mathbb{Z}^2 : 0 < a^2 + b^2 < R$. Compute

$$\lim_{R \to \infty} \sum_{(a,b) \in \square_R} \frac{(-1)^{a+b}}{a^2 + b^2}.$$

(Proposed by Rodrigo Angelo, Princeton University and Matheus Secco, PUC, Rio de Ja

Hint Sol

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