

IMC 2015, Blagoevgrad, Bulgaria

Day 2, July 30, 2015

Problem 6. Prove that

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(n+1)} < 2.$$

(10 points)

Problem 7. Compute

$$\lim_{A \rightarrow +\infty} \frac{1}{A} \int_1^A A^{\frac{1}{x}} dx.$$

(10 points)

Problem 8. Consider all 26^{26} words of length 26 in the Latin alphabet. Define the *weight* of a word as $1/(k+1)$, where k is the number of letters not used in this word. Prove that the sum of the weights of all words is 3^{75} .

(10 points)

Problem 9. An $n \times n$ complex matrix A is called *t-normal* if $AA^t = A^tA$ where A^t is the transpose of A . For each n , determine the maximum dimension of a linear space of complex $n \times n$ matrices consisting of t-normal matrices.

(10 points)

Problem 10. Let n be a positive integer, and let $p(x)$ be a polynomial of degree n with integer coefficients. Prove that

$$\max_{0 \leq x \leq 1} |p(x)| > \frac{1}{e^n}.$$

(10 points)